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CONTINUAL MODELING OF TWO-ROW FINITE DISCRETE SYSTEM DEFORMATION WITH REGARD FOR BOUNDARY EFFECTS

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A model in the form of a two-row finite discrete system that consists of some masses fastened by rods is constructed. Some dynamic motion equations are given for each mass. A field long-wave approximation is constructed with regard for boundary effects.

The replacement of nonhomogeneous systems by their homogeneous models is efficiently used for solving a number of problems. Latticed constructions are widely used in practice. As a rule, model corrections being made with regard for certain effects caused by the construction structure are based on rejecting some hypotheses accepted for the simplest models [1]. If we reject the assumption that a system motion can be completely determined by one displacement field and use several displacement fields for modeling, then we come to the problem on multifield models construction. In particular, some questions of multifield approach development are considered in [2, 3], and some questions of boundary operators construction are considered in [4].

As an example, in the present paper we consider a two-row finite discrete system. It is shown with the help of the multifield approach that some boundary layer rapidly oscillating displacement field appears near edges as well as in neighborhoods of defects and local loadings if the system is under extension. In particular, these models are of interest for in studying destruction processes.

1. DISCRETE EQUATIONS, EQUATIONS FOR ONE- AND TWO-FIELD MODELS

The equations for the transverse displacements v_n of two-row system parts (Figure 1) under action of a force f have the form:

$$
\begin{aligned}
&E_3B^2(v_{n+1}+2v_n+v_{n-1})/r+E_5v_n/(b/2)=f\,, \quad n=0,1,\dots,N\\
&v_{-1}+v_0=0\,, \quad v_N+v_{N+1}=0
\end{aligned}
\tag{1}
$$

if we neglect displacements along the system axis. Here $B=b/r$, $r=\sqrt{a^2+b^2}$, E_m are the rods rigidities.

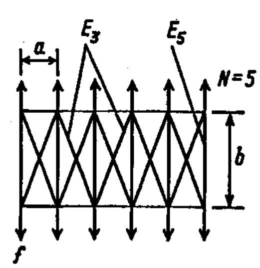

Fig. 1.

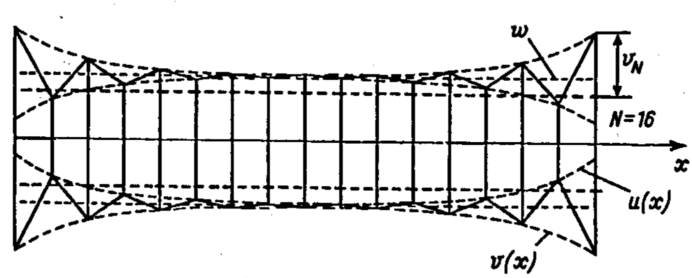

Fig. 2.

Denote $v(na)=v_n$. Then using the Taylor expansion in (1), we obtain the one-field model equation

$$
E_3B^2a^2v_{xx}/r+\left[2E_3B^3+E_5\right]v/(b/2)=f
\tag{2}
$$

For the two-field model we introduce two functions $u(x)$ and $v(x)$ such that $u((2m-1)a) = v_{2m-1}$, $v(2ma) = v_{2m}$. We perform the Taylor expansion of these two-field functions. If we retain the second derivatives, then we obtain the motion equations and the boundary conditions:

$$\begin{aligned}
&E_3B^3\left[v + a^2v_{xx}/2\right] + \left[E_3B^3 + E_5\right]u = (b/2)f\\
&\left[E_3B^3 + E_5\right]v + E_3B^3\left[u + a^2u_{xx}/2\right] = (b/2)f\\
&u(0) - au_x(0) + a^2u_{xx}(0)/2 + v(0) = 0\\
&u(l) + v(l) + au_x(l) + a^2u_{xx}(l)/2 = 0\,, \quad l = Na
\end{aligned} \tag{3}$$

2. SOLUTIONS COMPARISON

We look for a solution v_n of problem (1) in the form of some displacement in the rod middle part and a boundary layer solution $v_{0,n}$ as follows:

$$v_n = \begin{cases} w + v_{0,n}\,, & 0 \le n < n_0\\ w\,, & n_0 \le n \le N/2\\ \text{symmetrically for:} & N/2 \le n \le N \end{cases} \tag{4}$$

where $w = (b/2)(f/E)$, $v_{0,n} = (-1)^n\left(-2/[1-e^{\lambda}]\right)e^{-\lambda n w}$, $E = 2E_2B^3 - E_5$, $\lambda > 0$ is a solution of the equation $\cosh(\lambda) = 1 + c$, $c = E_5/(E_3B^3)$. The value n_0 determines a boundary effect propagation zone.

The homogeneous equation corresponding to equation (2) for the one-field model has no decreasing solution. If in (2) we neglect the term with v_{xx}, then we obtain the averaged equation $Ev/(b/2) = f$. Its solution coincides with w and can be used to determine displacements in domains remote from the boundaries.

Equations (3) for the two-field model have a particular solution $\overline{w}$; the corresponding homogeneous equations have boundary layer solutions $\overline{v}_0(x)$. Therefore, we can satisfy the boundary conditions and represent the solution $\overline{v}(x)$ of (3) in the form

$$\overline{v}(x) = \begin{cases} \overline{w} + \overline{v}_0(x)\,, & 0 \le x < x_0\\ \overline{w}\,, & x_0 \le x \le l/2\\ \text{symmetrically for:} & l/2 \le x \le l \end{cases} \tag{5}$$

where

$$\overline{w} = w\begin{bmatrix}1\\1\end{bmatrix}, \quad \overline{v}_0(x) = \frac{-2}{1-[1+\mu+\mu^2/2]}e^{-\mu x/a}w\begin{bmatrix}-1\\1\end{bmatrix}, \quad \mu = \sqrt{2c}$$

The value x_0 determines a boundary effect propagation zone.

In domains remote from the boundaries, discrete solution (4) and two-field solution (5) coincide for $x = na$. In domains closed to the boundaries, the closeness of these solutions for $x = na$ is determined by the coefficient $\mu = \sqrt{2c}$ that is a root of the equation $\mu^2 - 2c = 0$.

The example that illustrates the closeness of the discrete and two-field solutions is represented in Figure 2. It is seen from Figure 2 that two fields introduced here to describe displacements reflect the following (characteristic from the physical viewpoint) feature of the deformable system state: the rods are divided into two groups — compressed and extended with respect to the averaged displacements w. Some questions of mathematical justification for the multifield approach in modeling of boundary effects are considered in [5].

Note in conclusion that the given system is a model for a number of macro- and microsystems; in particular, it can be considered as the simplest structural model of an interlayer for destruction modeling under separation.

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