MULTI-FIELD MODELLING OF SHORT WAVELENGTH DEFORMATIONS FOR COSSERAT SOLIDS

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Summary We propose the method and derive the hierarchical system of multi-field models, which describe the elastic properties of Cosserat lattice with an increasing accuracy. We show that the multi-field models are valid not only for long but also for short waves and make it possible to obtain localized rapidly varying static deformations.

INTRODUCTION

The classical Cosserat continuum model has wide applications in problems where not only displacements, but also rotations of structural elements should be taken into account. Rotational degrees of freedom naturally appear, for example, for bodies with beam-like microstructure, or with elements having finite sizes. However, it is often stated that short wavelength deformations are inaccessible for modelling within the framework of continuum models. Meanwhile, such deformations may take place in structural solids near boundaries, defects, localised forces, and may become very important in fracture, instability, and plasticity problems.

MULTI-FIELD MODELS

We consider a square Cosserat lattice, i.e. lattice, deformations of which are determined not only by displacements u_n, v_n, but also by rotations φ_n of elements. The single-field micropolar model can be obtained on the basis of discrete equations of motion for particles of the elementary cell by using the single vector function $\{u(x,y,t),\ v(x,y,t),\ \varphi(x,y,t)\}$ for describing displacements and rotations of the elements of the lattice. It is necessary to note that it is not self-evident assumption of the single-field theory. For deriving the *N*-field model we consider as a basis a macrocell, which consists of *N* elementary cells. Although, all elements of the lattice (Fig. 1a) are identical they are marked with different numbers (an examples are shown in Fig. 1b-e). We use the notations $u_{k,m}^{[n]}$, $v_{k,m}^{[n]}$, $\varphi_{k,m}^{[n]}$ with superscript $n=\overline{1,N}$ for the components of vector of generalised displacements. Then, for the particles marked by different numbers we obtain $3N$ discrete equations of motion. Instead of using the single vector function, *N* vector functions $\{u^{[n]}(x,y,t),\ v^{[n]}(x,y,t),\ \varphi^{[n]}(x,y,t)\}$ are used in the *N*-field theory to describe the displacements and rotations of particles marked by numbers $n=\overline{1,N}$, respectively. By using Taylor series expansions of displacements and rotations in the discrete equations around the points at which the equations are written, we come to $3N$ equations of *N*-field theory. Keeping derivatives up to $K>2$ order in the equations, we come to higher-order gradient continuum models.

RESULTS AND DISCUSSION

We consider a lattice with the potential energy of connections between neighbouring elements, which is used in models of granular media [1, 2]. It generalises the potential energy of a beam finite element, which is often used in lattice models of constructions and materials with beam-like microstructure. The single-field model is constructed by using the simple cell ($N=1$). This leads to the conventional Cosserat model for $K=2$ [1] or to the higher-order gradient micropolar model in the case $K=4$ [2]. By using the procedure described above, we derive three types of the two-field model, $N=2$, corresponding to macro-cells presented in Fig. 1b-d. In the linear case, the derived systems of six equations can be split in two uncoupled systems. One of them is the system of the equations of the micropolar theory and, therefore, all models possess properties of conventional micropolar model and are valid for modelling of slowly varying deformations. The second system varies with the model. Its meaning is clarified by the analysis. The derivation of the continuum models starting from structural model gives us the possibility to test the accuracy and compare their properties. We derive dispersion relations for plane waves by using the lattice and multi-field models. The comparison shows that the models “b”, “c”, and “d” specify the single-field micropolar model for short waves with the wave numbers around the corners $(0,\pi)$, (π,π), and $(\pi,0)$ of the first Brillouin zone, respectively. The four-field model (Fig. 1e) combines the conventional Cosserat model and two-field models into one, which possesses the dynamical properties of these models and can be applied for modelling of both long and short wavelength deformations [3].

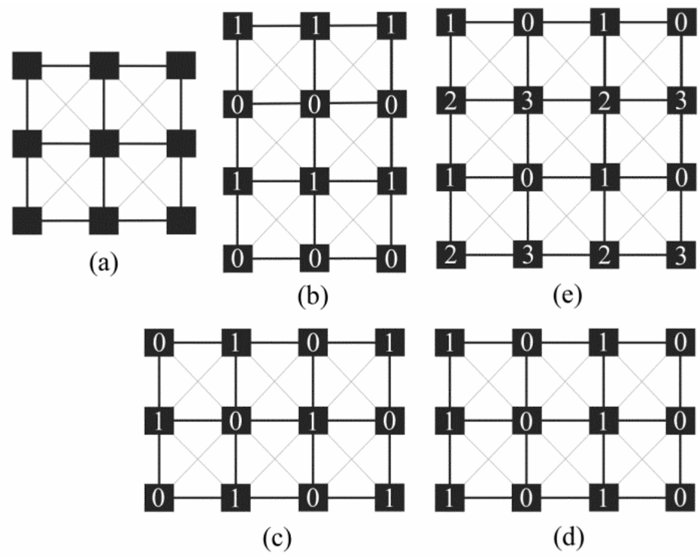

Fig. 1 a-e.

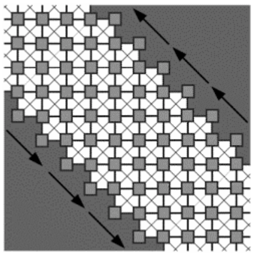

Fig. 2.

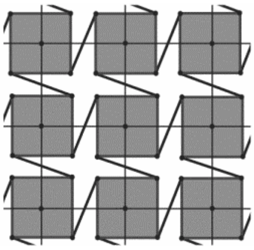

Fig. 3.

Higher-order gradient multi-field micropolar models ($K = 4$) are constructed by taking into account higher-order gradient terms in the equations. The comparison with the models with derivatives up to the second order ($K = 2$) shows that higher-order gradient models improve the approximation in the area of long waves and for short wavelength waves near the corresponding points of the first Brillouin zone.

The comparative analysis of the models in the description of spatially localised static one-dimensional deformations of discrete system is carried out for one-dimensional deformations of a lattice in problem of tension-compression and its shear (see Fig. 2) between parts, which are considered as rigid. We demonstrate an example of the short wavelength spatially localised static deformations, which could not be obtained in the frame of the classical single-field approach, but are constructed by using the derived multi-field model.

We demonstrate that proposed technique may be implemented for modelling of innovative materials with unusual properties. We derive multi-field models, find macroscopic parameters, and illustrate above mentioned properties on the examples of auxetic materials, i.e., materials having negative Poisson’s ratio, by considering generalised Ishibashi-Iwata model [4, 5] and materials with chiral microstructure (see Fig. 3).

CONCLUSION

By using of a square lattice of elements with rotational degrees of freedom, we obtain the hierarchical system of continuum models, which describe the dynamical properties of the lattice with increasing accuracy. We utilize ideas and methods of micropolar and higher-order gradient theories to develop the multi-field theory. We show that their applications allow to describe qualitatively different effects of solids with microstructure. By increasing the number of fields, the multi-field approach gives a natural way to describe both long- and short wavelength deformations within the framework of generalised continuum mechanics.

References

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